Retrospective blind motion correction of MR images

Alexander Loktyushin[1], Hannes Nickisch[2], Rolf Pohmann[3], Bernhard Schölkopf[1]

[1]MPI for Intelligent Systems, Tübingen, Germany; [2]Philips Research Laboratories, Hamburg, Germany; [3]MPI for Biological Cybernetics, Tübingen, Germany

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Contribution

- Patient motion in the scanner is an unsolved problem in MRI
- We propose a new retrospective method to motion correction
- Our fully analytic forward model allows efficient gradient-based exploration of space of motion parameters
- No tracking devices or specialized sequences are required
- Our approach was tested on real data (TSE, FLASH, MPRAGE)

Our method

Key aspects:

- Retrospective method
- No need for tracking devices
- No specialized sequences are required
- Gradient-based optimization

We assume:

- Scanned object is a rigid body
- No strong intra-view motion
- Object is not leaving the Field of View
- Raw k-space data is available

General framework:

Let $\mathbf{F} \in \mathbb{C}^{n\times n}$ be the orthonormal Fourier matrix,
$\mathbf{u} \in \mathbb{C}^n$ the unknown sharp image of size $n = n_1 \cdot n_2 \cdot n_3$ pixels,
$\mathbf{M} = \mathrm{dg}(\mathbf{m}) \in [0,1]^{n\times n}$ where $\mathbf{m} \in [0,1]^n$ is a diagonal masking matrix,
$\tilde{\mathbf{A}}_{\boldsymbol\theta}$ is a general rigid motion transformation matrix,
$\mathbf{A}_{\boldsymbol\theta} \in \mathbb{C}^{n\times n}$ is a matrix such that $\mathbf{F}\tilde{\mathbf{A}}_{\boldsymbol\theta} = \mathbf{A}_{\boldsymbol\theta}\mathbf{F}$,
$\boldsymbol\theta_t \in \mathbb{R}^3 \times [0, 2\pi)^3$ is the vector with translation and rotation motion parameters at time t.
Assuming additive Gaussian noise $\boldsymbol\varepsilon \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$, the acquisition in k-space can be written as a noisy linear process

$$\mathbf{y} = \int_0^T \mathrm{dg}(\mathbf{m}_t)\mathbf{A}_{\boldsymbol\theta_t}\mathrm{d}t\,\mathbf{F}\mathbf{u} + \boldsymbol\varepsilon \in \mathbb{C}^n,\ \mathbf{1} = \int_0^T \mathbf{m}_t \mathrm{d}t. \quad (1)$$

with $\mathbf{M}_t$ selecting the segment in k-space being filled by the scanner at time t.
In a real setting of Cartesian acquisitions with finite number of views, the measurement integral (1) becomes a sum over the number of excitations and the masking becomes binary instead of continuous $\mathbf{M} = \mathrm{dg}(\mathbf{m}) \in \{0,1\}^{n\times n}$ where $\mathbf{m} \in \{0,1\}^n$

$$\mathbf{y} = \sum_{t=1}^{T} \mathrm{dg}(\mathbf{m}_t)\mathbf{A}_{\boldsymbol\theta_t}\mathbf{F}\mathbf{u} + \boldsymbol\varepsilon \in \mathbb{C}^n,\ \mathbf{1} = \sum_{t=1}^{T}\mathbf{m}_t. \quad (2)$$

Assume for now that the k-space lines of an image of size n_1 by n_2 pixels are measured from top to bottom, hence $T = n_1$ steps are needed. Then, in every repetition, a noisy version of $\mathrm{dg}(\mathbf{m}_t)\mathbf{A}_{\boldsymbol\theta_t}\mathbf{F}\mathbf{u} \in \mathbb{C}^n$ is measured. But since $\mathbf{m}_t$ is binary this is equivalent to say that we acquire a noisy version of n_2 components instead of $n = n_1 n_2$, which we denote by $[\mathbf{A}_{\boldsymbol\theta_t}]_{\mathbf{m}_t}\mathbf{F}\mathbf{u} \in \mathbb{C}^{n_2}$ with the shortcut $[\mathbf{A}_{\boldsymbol\theta_t}]_{\mathbf{m}_t} \in \mathbb{C}^{n_2\times n}$

$$\mathbf{y} = \mathbf{A}_{\boldsymbol\theta}\mathbf{F}\mathbf{u} + \boldsymbol\varepsilon \in \mathbb{C}^n,\ \mathbf{A}_{\boldsymbol\theta} := \begin{bmatrix} [\mathbf{A}_{\boldsymbol\theta_1}]_{\mathbf{m}_1} \\ [\mathbf{A}_{\boldsymbol\theta_2}]_{\mathbf{m}_2} \\ \vdots \\ [\mathbf{A}_{\boldsymbol\theta_T}]_{\mathbf{m}_T} \end{bmatrix} \in \mathbb{C}^{n\times n}. \quad (3)$$

with $\boldsymbol\theta \in \mathbb{R}^{3T} \times [0, 2\pi)^{3T}$ being the vector with all the motion parameters of the trajectory, and $T = n_2 \cdot n_3$. The bottom line is that a matrix vector multiplication (MVM) with $\mathbf{A}_{\boldsymbol\theta}$ can be done in $\mathcal{O}(n)$ time since $\mathbf{A}_{\boldsymbol\theta}$ can be decomposed into blocks whose MVMs can be performed efficiently.

Objective function and derivatives

As an estimator of the image quality we use the entropy of the gradients of the image metric ϕ, that according to McGee et al. (2000) is the best metric for medical images:

$$\phi(\mathbf{u}) = H(\mathbf{D}_x\mathbf{u}) + H(\mathbf{D}_y\mathbf{u}) \quad (4)$$

where $\mathbf{D}_x$ and $\mathbf{D}_y$ are horizontal and vertical finite difference matrices, and $H(\cdot)$ is the entropy estimator defined as:

$$H(\mathbf{u}) = \mathbf{v}^\top \ln \mathbf{v}, \mathbf{v} = \frac{|\mathbf{u}|}{\|\mathbf{u}\|} \in \mathbb{R}^n_+, |\mathbf{u}| = \sqrt{\Im^2(\mathbf{u}) + \Re^2(\mathbf{u})} \in \mathbb{R}^n_+, \|\mathbf{u}\| = \sqrt{\mathbf{u}^{\mathsf{H}}\mathbf{u}} \in \mathbb{R}_+.$$

Using our objective function (5), we seek the motion parameters θ, that invert the motion in observed image $\mathbf{y}$ in such a way, that the focus criterion ϕ for the corrected image in spatial domain is optimized:

$$\hat{\boldsymbol\theta} = \arg\min_{\boldsymbol\theta}(\phi(\mathbf{F}^{\mathsf{H}}\mathbf{A}_{\boldsymbol\theta}\mathbf{y})), \mathbf{u} := \mathbf{F}^{\mathsf{H}}\mathbf{A}_{\hat{\boldsymbol\theta}}\mathbf{y} \quad (5)$$

The derivatives are of the form

$$\frac{\partial\phi(\mathbf{A}_{\boldsymbol\theta}\mathbf{u})}{\partial\boldsymbol\theta} \in \mathbb{R}^n$$

where $\mathbf{A}_{\boldsymbol\theta} \in \mathbb{C}^{n\times n}$ is a matrix with parameter $\boldsymbol\theta \in \mathbb{R}^n$ such that every row i depends on θ_i only, $\mathbf{u} \in \mathbb{C}^n$ a vector and $\phi : \mathbb{C}^n \to \mathbb{R}$ is a continuously differentiable function. We denote by $\mathbf{A}'_{\boldsymbol\theta}$ the matrix such that $[\frac{\partial}{\partial\theta_i}\mathbf{A}_{\boldsymbol\theta}\mathbf{u}]_i = [\mathbf{A}'_{\boldsymbol\theta}\mathbf{u}]_i$ i.e. $\mathbf{A}'_{\boldsymbol\theta}$ contains all the information needed for the Jacobian

$$\frac{\partial\mathbf{A}_{\boldsymbol\theta}\mathbf{u}}{\partial\boldsymbol\theta^\top} = \mathrm{dg}(\mathbf{A}'_{\boldsymbol\theta}\mathbf{u}), \quad (6)$$

which is diagonal because every row i depends on θ_i only. Now, let $\frac{\partial\phi(\mathbf{v})}{\partial\mathbf{v}}$ denote the gradient of the objective function $\phi : \mathbb{C}^n \to \mathbb{R}$. Then, the desired derivative $\partial\phi(\mathbf{A}_{\boldsymbol\theta}\mathbf{u})/\partial\boldsymbol\theta$ can be obtained via the chain rule

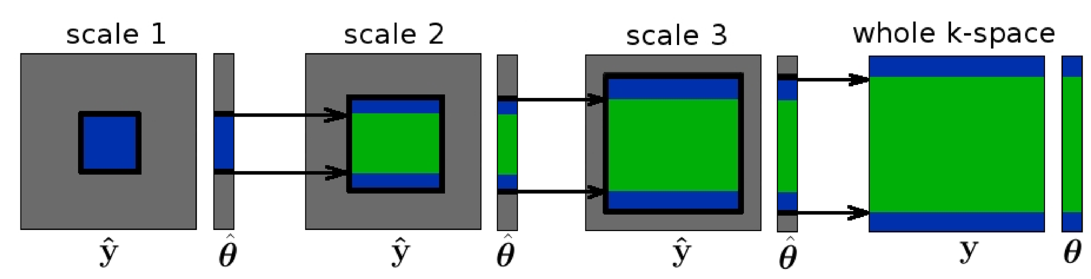
$$\frac{\partial\phi(\mathbf{A}_{\boldsymbol\theta}\mathbf{u})}{\partial\boldsymbol\theta} = \frac{\partial(\mathbf{A}_{\boldsymbol\theta}\mathbf{u})^\top}{\partial\boldsymbol\theta}\frac{\partial\phi(\mathbf{v})}{\partial\mathbf{v}} = (\mathbf{A}'_{\boldsymbol\theta}\mathbf{u}) \odot \left(\frac{\partial\phi(\mathbf{v})}{\partial\mathbf{v}}\right). \quad (7)$$

Multiscale optimization

Our objective is a highly non-linear function. We solve the problem of local minima by using a multi-scale coarse-to-fine approach:

Input: Corrupted volume $\mathbf{y}$ with $n_1 \cdot n_2 \cdot n_3$ complex coefficients in frequency domain, with DC component at $[c_1, c_2, c_3] = \left[\frac{n_1}{2}+1, \frac{n_2}{2}+1, \frac{n_3}{2}+1\right]$. Also, assume $n_3 = n_2$.
Output: Restored volume $\mathbf{u}$ in spatial domain

for $s \leftarrow \frac{64}{2}, \dots, \frac{n_2}{2}$ **do**
  $\hat{\mathbf{y}} \leftarrow \mathbf{y}([c_1, c_2, c_3] - s : [c_1, c_2, c_3] + s)$;
  Calculate $\hat{\boldsymbol\theta} \leftarrow \arg\min_{\boldsymbol\theta}(\phi(\mathbf{F}^{\mathsf{H}}\mathbf{A}_{\boldsymbol\theta}\hat{\mathbf{y}}))$;
  Initialize central frequency part of $\hat{\boldsymbol\theta}$ on next finer scale with $\hat{\boldsymbol\theta} \leftarrow \boldsymbol\theta([c_2, c_3] - s : [c_2, c_3] + s)$;
end
Finally, obtain the sharp image: $\mathbf{u} := \mathbf{F}^{\mathsf{H}}\mathbf{A}_{\hat{\boldsymbol\theta}}\mathbf{y}$;



We regularize the recovered motion parameters by putting L2 penalty on the difference of consecutive motion parameters, which helps to avoid strong spikes in the recovered trajectory, which are often the artifact of the algorithm:

$$\varphi(\boldsymbol\theta) := \phi(\mathbf{A}_{\boldsymbol\theta}\mathbf{F}^{\mathsf{H}}\mathbf{y}) + \lambda\rho(\mathbf{D}\boldsymbol\theta) \quad (8)$$

Implementation of $\mathbf{A}_{\boldsymbol\theta}$ and run-time

Translation:

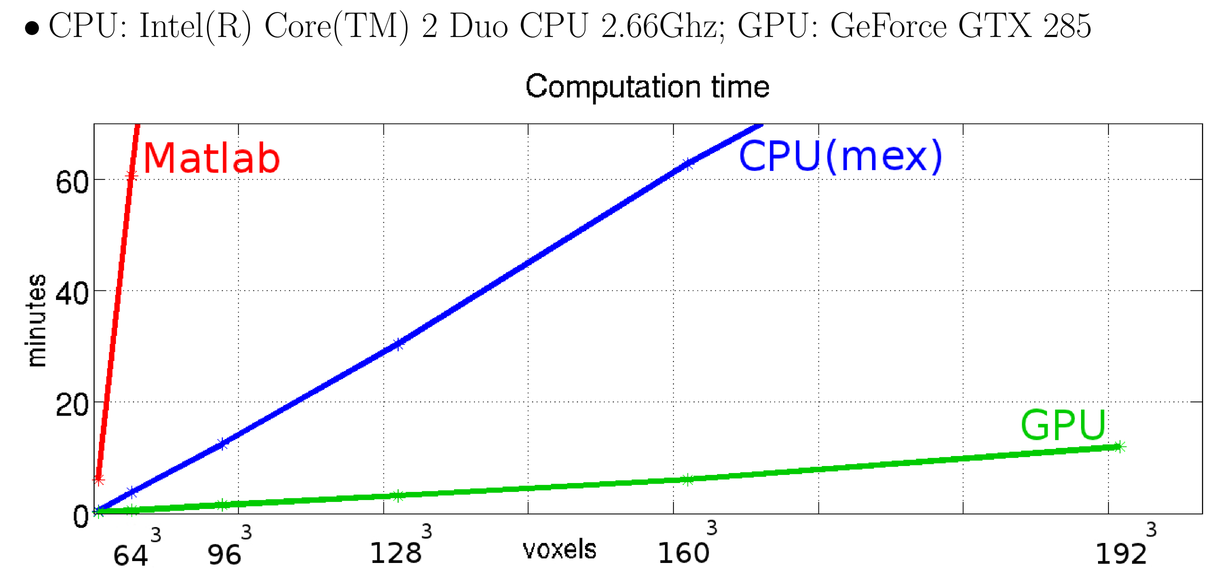
- The matrix is diagonal with entries of the form $e^{-2\pi i\boldsymbol\theta_t\xi_j}$, where $\boldsymbol\theta_t$ is the spatial displacement, ξ_j is phase ramp, and j is the frequency index

Rotation:

- The matrix contains interpolation weights
- Given the rotation angles the coordinates of the knots on rotated lines are found
- Interpolation is done over 4^d knot neighbours, where d is the number of dimensions
- Cubic kernel is used as an interpolating function

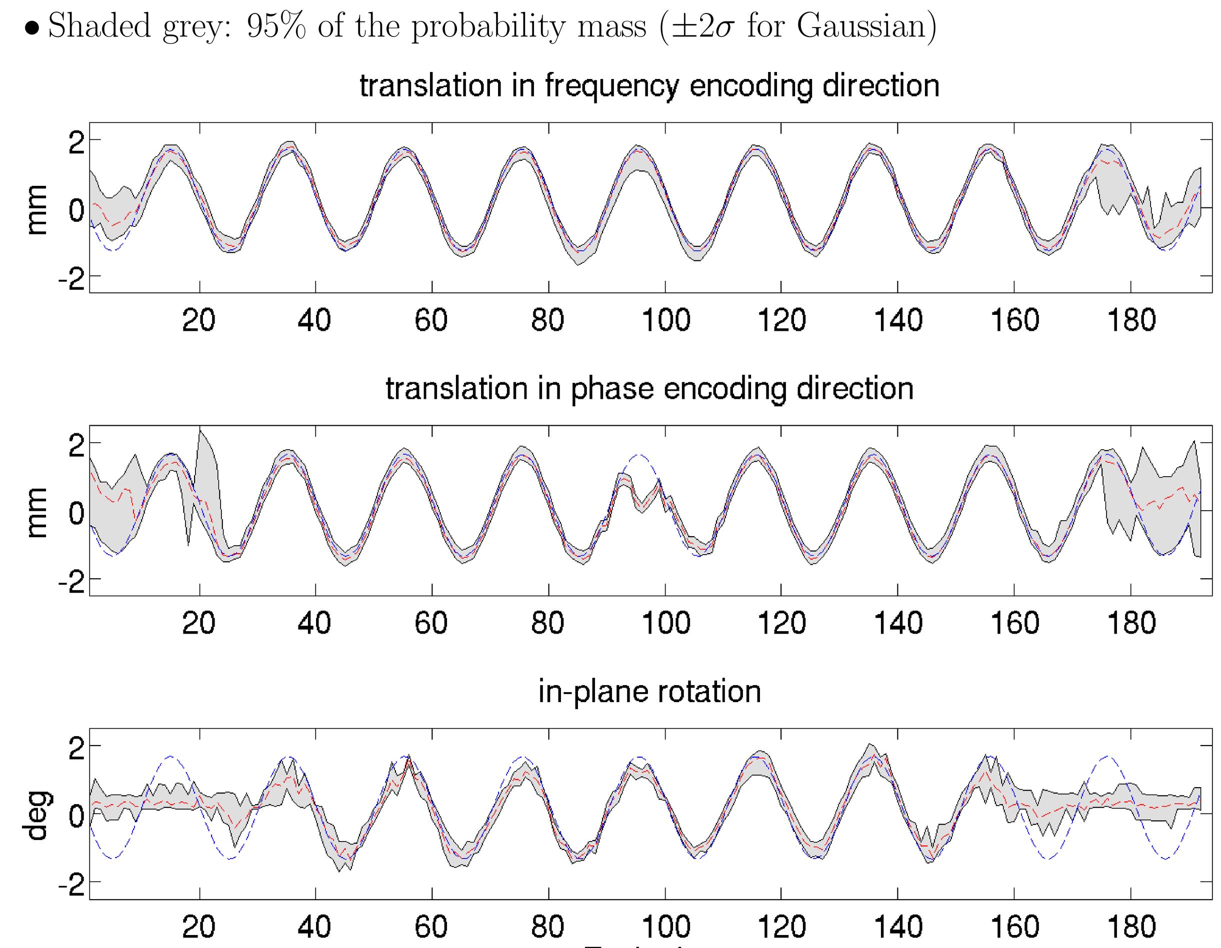
Optimization:

- We use the LBFGS non-linear optimizer (50 function evaluations per scale)
- The computational bottleneck are the Fast Fourier transforms costing $\mathcal{O}(n \log n)$, both $\mathbf{A}_{\boldsymbol\theta}$ and its derivatives are of $\mathcal{O}(n)$ complexity
- CPU: Intel(R) Core(TM) 2 Duo CPU 2.66Ghz; GPU: GeForce GTX 285



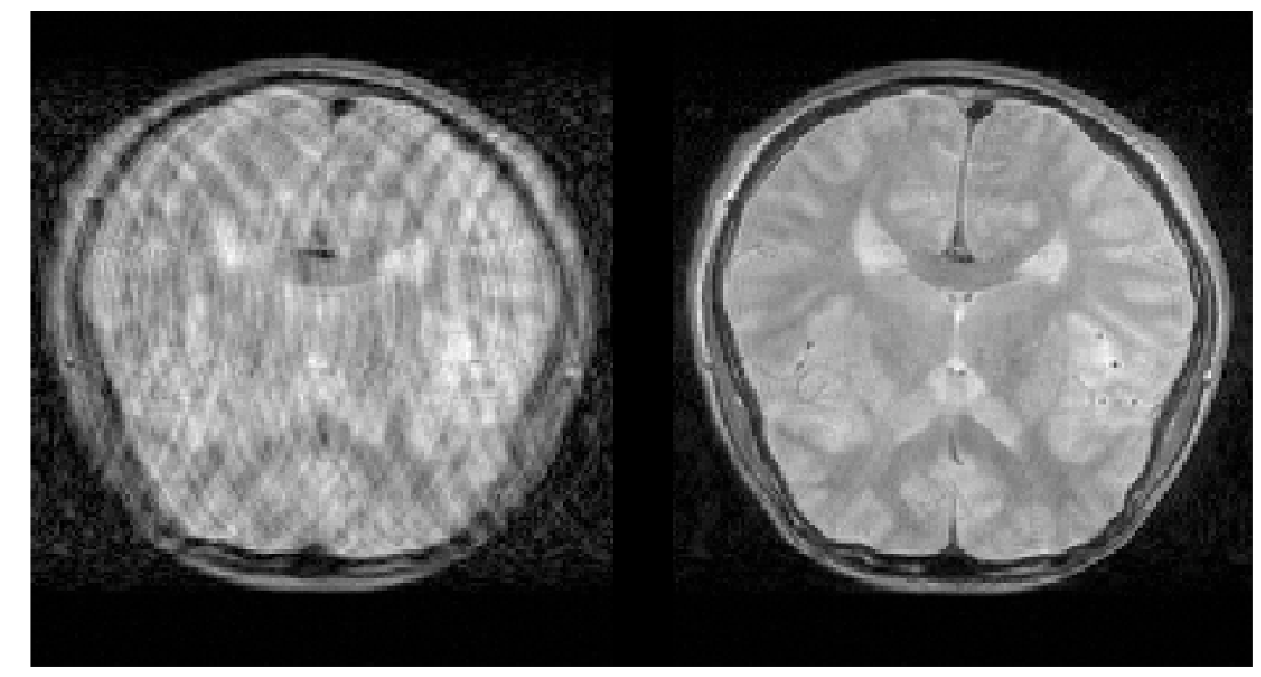
Experiments: simulation (2D)

- 100 trials were made to test the stability of our reconstruction algorithm
- On each trial the algorithm started from randomly initialized motion parameters
- Dashed blue: ground truth sinusoidal trajectory
- Dashed red: empirical mean over trajectories recovered by the algorithm
- Shaded grey: 95% of the probability mass ($\pm 2\sigma$ for Gaussian)

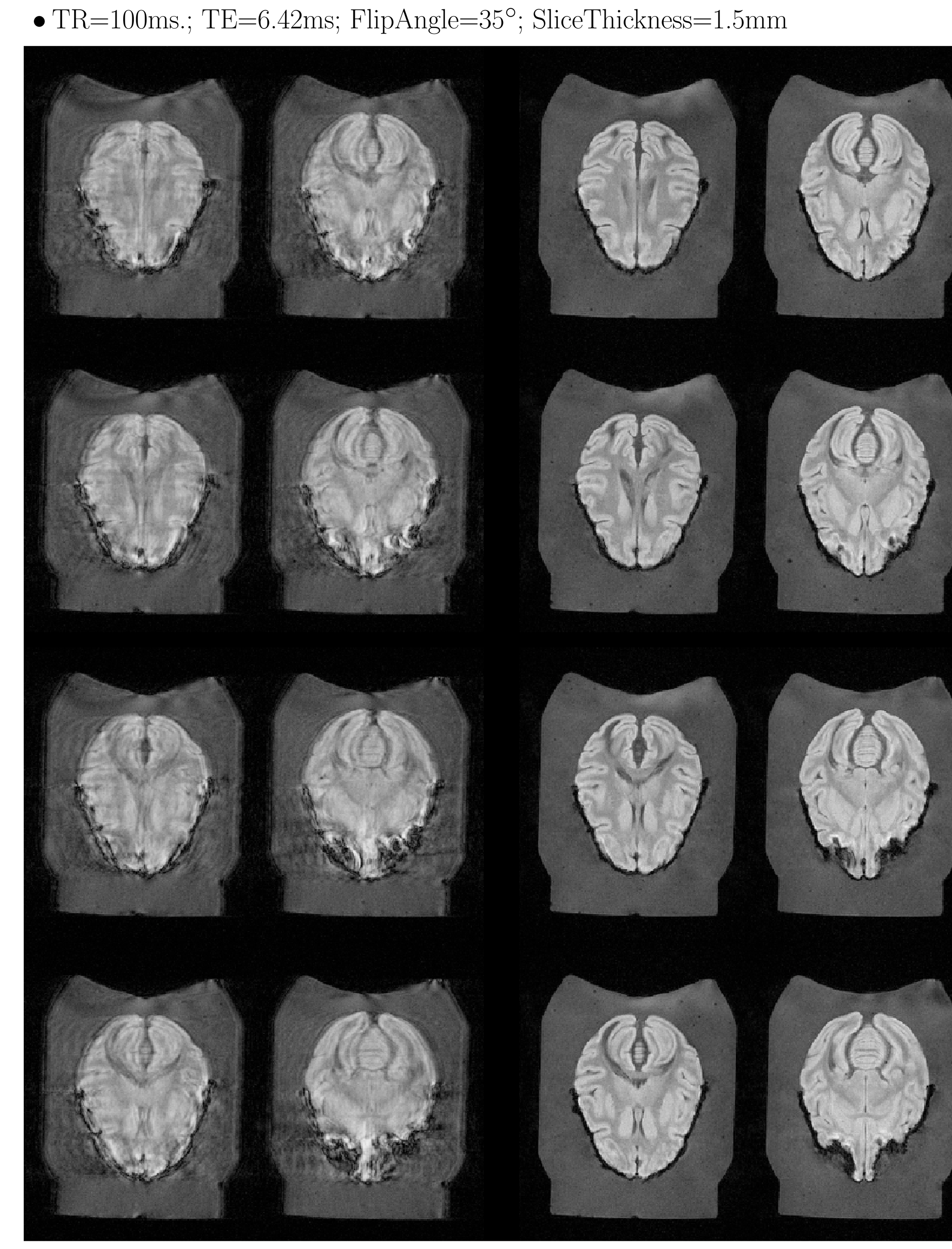


Experiments: real data (2D+3D)

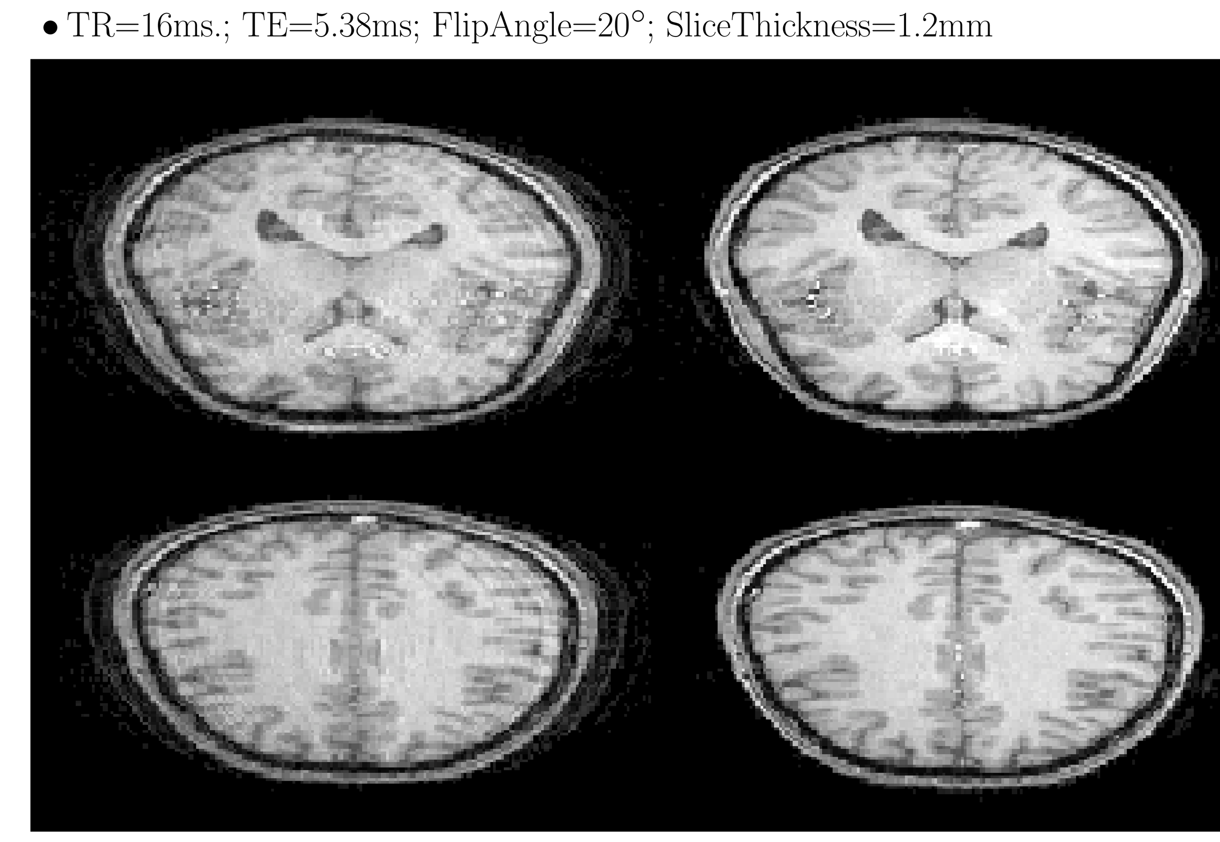
- Freely moving human subject
- 2D image (384x160) acquired with TSE sequence; Siemens 3T Trio scanner
- TR=3500ms.; TE=40ms; FlipAngle=180°; SliceThickness=4mm



- Monkey's brain in fixation gel; controlled motion
- 3D volume (384x192x16) acquired with FLASH sequence
- TR=100ms.; TE=6.42ms; FlipAngle=35°; SliceThickness=1.5mm



- Freely moving human subject
- 3D volume (384x192x96) acquired with FLASH sequence
- TR=16ms.; TE=5.38ms; FlipAngle=20°; SliceThickness=1.2mm



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