Optimization of k-Space Trajectories by Bayesian Experimental Design

MAX-PLANCK-GESELLSCHAFT

Matthias Seeger[1,2], Hannes Nickisch[2], Rolf Pohmann[2] and Bernhard Schölkopf[2]

[1]Saarland University, Saarbrücken, Germany

[2]Max-Planck-Institute for Biological Cybernetics, Tübingen, Germany

Max-Planck-Institute for biological Cybernetics

Compressed Sensing in MRI:

Compressed Sensing uses low-level statistical properties of MR-images to reconstruct images from undersampled k-space data (1). It takes advantage of the *sparsity* of the image or its wavelet or finite difference domain, a property that distinguishes (MR-)images from noise and can be assumed to apply to any MR image. While much has been done to optimize the reconstruction algorithms, the question of how to optimally sample k-space for sparse reconstruction has not been addressed before. Here, Bayesian inference is used for a directed search for the optimal k-space trajectories. Using a new convex optimization algorithm for approximate inference, this approach is scaled up to full-size MR images.

The optimization algorithm is based on Bayes' theorem:

$$P(u \mid y) \propto P(y \mid u) \cdot P(u)$$

Posterior: Combines data and prior knowledge

Likelihood: Optimized in linear reconstruction

Prior: Enforces sparsity

with $y = Xu$

u **: imaged object**

y **: measured k-space data**

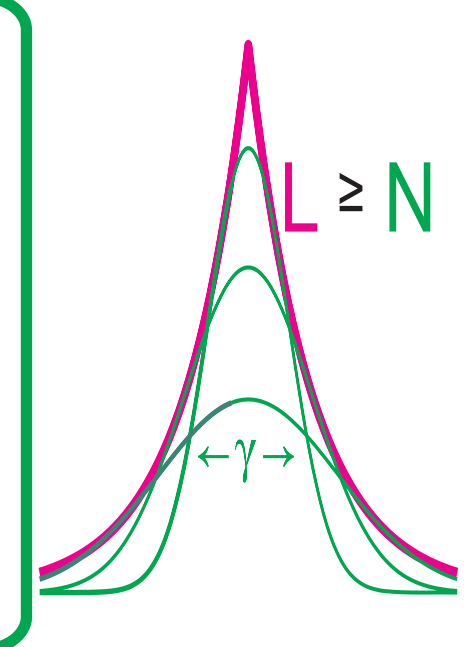
X: measurement matrix

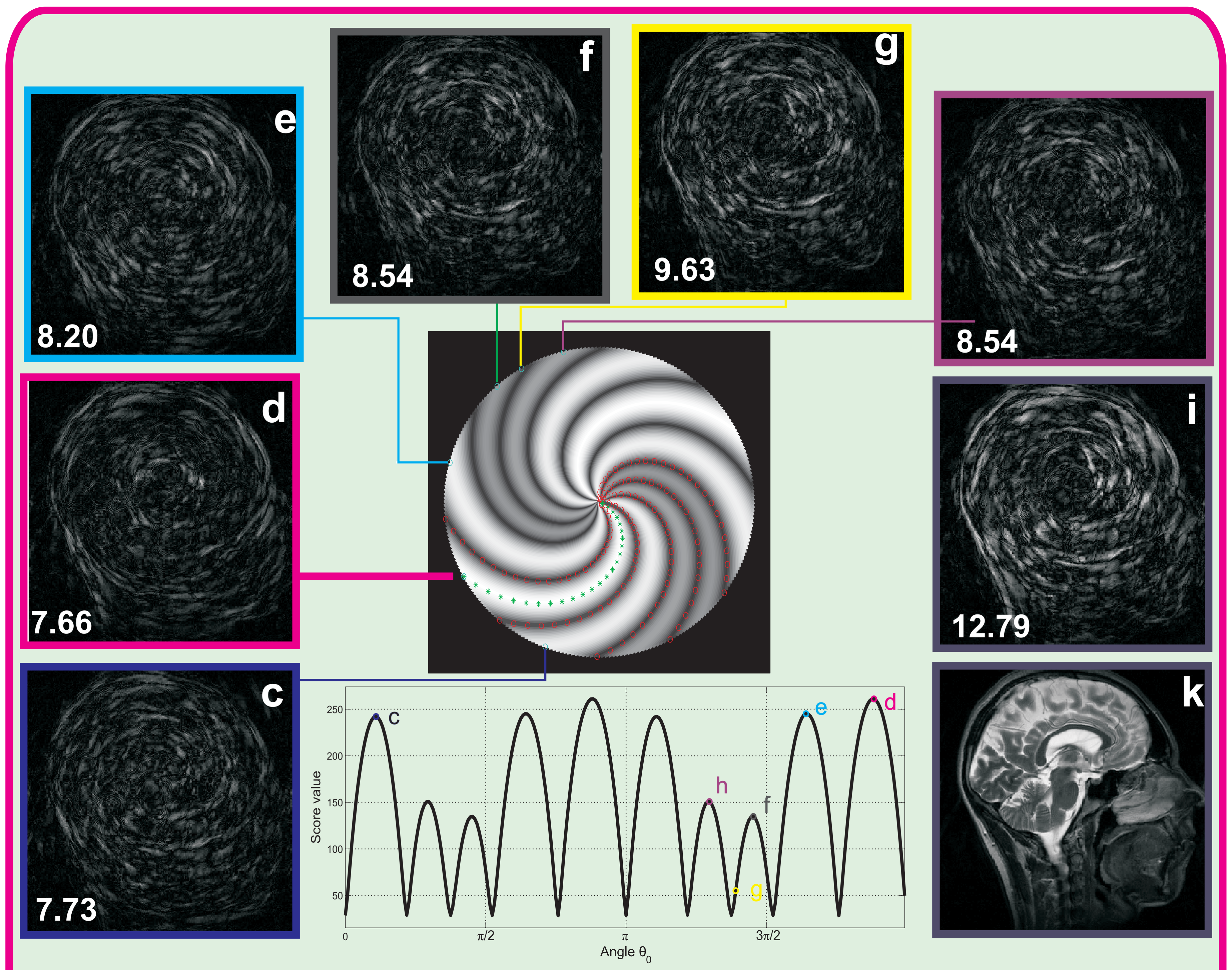
While standard linear reconstruction maximizes the likelihood, sparse reconstruction optimizes the posterior, thus including the constraint of sparsity.

Our algorithm focuses on the measurement design for sparse reconstruction by changing the measurement and thus the data *y*: Starting from an initial measurement matrix X, the expected information gain is computed for adding an additional k-space line. Different possible k-space trajectories are scored according to their reduction in uncertainty in the posterior:

Algorithm:

- Calculate posterior distribution
  The super-gaussian (Laplace) distribution is fitted by a Gaussian with optimized width as lower bound (s. right).
- Compute score values for all possible new k-space lines
  The score value estimates the expected information gain by minimizing the entropy of the posterior distribution.
- Take the one that minimizes posterior uncertainty and add it to the measurement matrix
- Acquire the new encode step
- Calculate new posterior
- Start again for next phase encode step

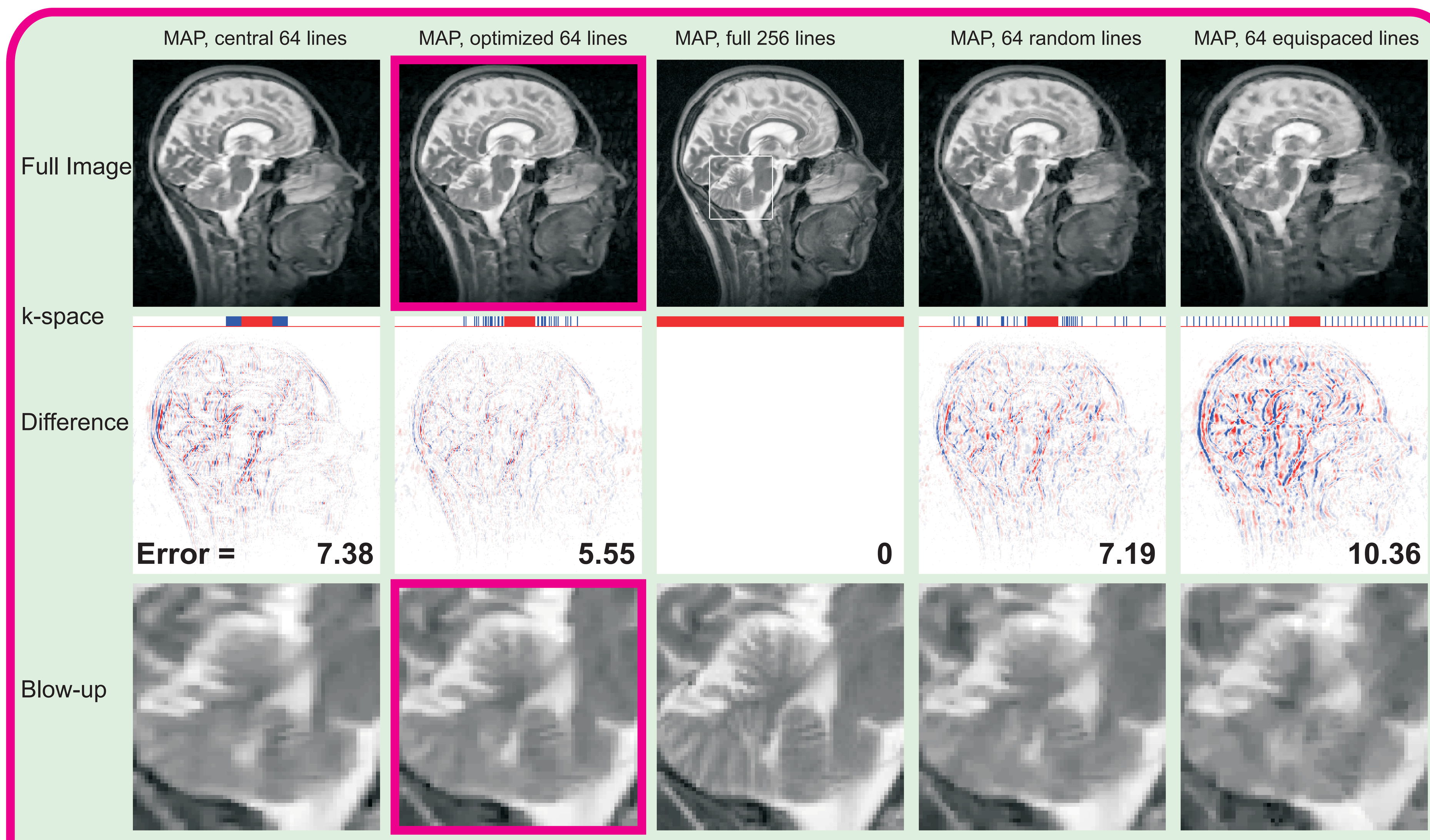




Demonstration of the optimization process on spiral trajectories:
(k): Full image reconstructed from 16 spirals.
(i): Difference image reconstructed from 5 spirals. The algorithm calculates score values for 256 spirals (bottom graph).
(c-h): Difference images for 6 possibilities for the sixth spiral, with L_2-errors. The selected trajectory (highest score value) is d, which also leads to the image with the smallest error.

Results:

TSE-images of the heads of five different subjects were acquired with a matrix size of 256×256 voxels for 16 slices with two different slice orientations and two echo times each. The MAP-estimation (1) was used for sparse reconstruction of images from a subset of these data for four different scenarios (center of k-space, equidistant k-space lines, random distribution and optimized), as well as from the full dataset for reference. Our Bayesian algorithm was used for the optimized setting.



MAP reconstructions for Cartesian undersampled data for 64 of 256 acquired lines. Four different k-space schemes, each consisting of the central 32 lines plus 32 lines positioned by different scenarios are compared to the fully sampled image (center). The given error is the squared difference between the fully sampled and the undersampled images. The optimized k-space scheme shows significantly better recovery of detail than images reconstructed from other undersampled data.

From four-fold undersampled images, the MAP-reconstruction was able to form images with good detail as compared to the fully sampled images. The image fidelity, quantified by the L_2-difference between undersampled and fully sampled image, is significantly reduced in the optimized version. The k-space scheme optimized for one image also gave best results when applied to different slice positions, volunteers, slice orientations or echo times, thus proving the robustness of the approach.

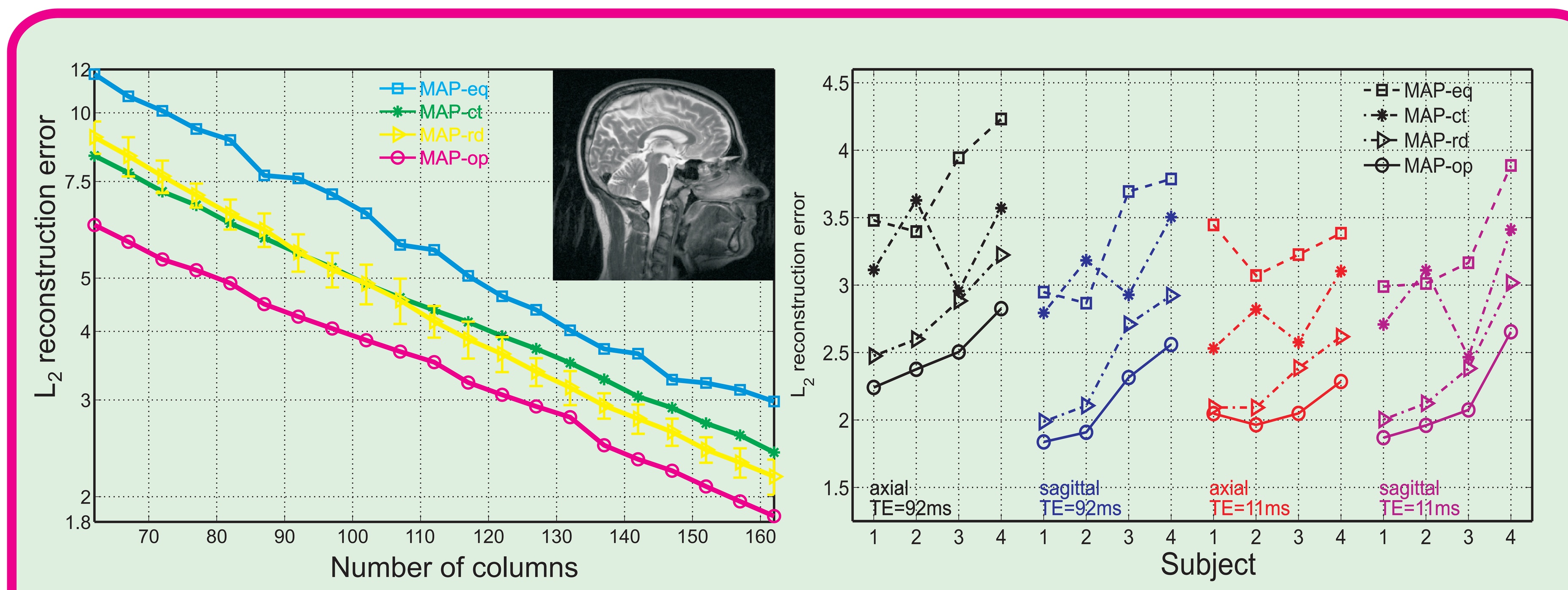


Image errors using the four different k-space schemes.
Left: For different numbers of k-space columns. Errors are integrated over all volunteers, slices, orientations and echo times and show a clear advantage with the optimized k-space coverage.
Right: Plotted separately for four subjects, each with two slice orientations and echo times. The optimized scheme minimizes the errors for all images, thus proving the robustness of the approach.

Conclusion:

Sparse reconstruction of MR-images can be substantially improved by optimizing the k-space coverage of the acquisition. Our procedure has shown to be stable, flexibly applicable to different experiments and scalable to the sizes of high-resolution MR images. In addition, optimization results are also valid for different (but similar) images, like other slices, subjects, echo times or slice orientations.

Reference:
(1) M. Lustig, D. Donoho, J. Pauly: *Sparse MRI: The Application of Compressed Sensing for Rapid MR Imaging.* MRM **58**, 1182-1195 (2007)