Our objective is a highly non-linear function. We solve the problem of local minima by using a multi-scale coarse-to-fine approach.

**Input:** Corrupted volume $v$ with $n_1 \times n_2 \times n_3$ complex coefficients in frequency domain, with DC component $A_0 = [c_0, c_0, c_0] + [n_1 + 1, n_2 + 1, n_3 + 1]$. Also, assume $n_1 = n_2$.

**Output:** Restored volume $u$ in spatial domain.

We regularize the recovered motion parameters by putting L2 penalty on the difference of consecutive motion parameters, which helps to avoid strong spikes in the recovered trajectory, which are often the artifact of the algorithm:

$$
\phi(\theta) = \langle A_0 \hat{u}(\theta) \rangle + \lambda \|D\theta\|_2
$$

### Implementation of $A_0$ and run-time

**Translation:**
- The matrix is diagonal with entries of the form $e^{-2\pi i \theta t}$, where $\theta$ is the spatial displacement, $\xi$ is phase ramp, and $\nu$ is the frequency index.

**Rotation:**
- The matrix contains interpolation weights.
- Given the rotation angle the coordinates of the knots on rotated lines are found.
- Interpolation is done over $4^2$ knot neighbours, where $m$ is the number of dimensions.
- Cubic kernel is used as an interpolating function.

### Optimization

- We use the L-BFGS non-linear optimizer (50 function evaluations per scale).
- The computational bottleneck are the Fast Fourier transforms costing $O(n \log n)$, both $A_0$ and its derivatives are of $O(n)$ complexity.
- CPU: Intel(R) Core(TM) 2 Duo CPU 2.66GHz. GPU: GeForce GTX 285

### Experiments: simulation (2D)

- 100 trials were made to test the stability of our reconstruction algorithm.
- On each trial the algorithm started from randomly initialized motion parameters.
- Dashed blue: ground truth sinusoidal trajectory.
- Dashed red: empirical mean over trajectories recovered by the algorithm.
- Shaded grey: 95% of the probability mass $s$ (for Gaussians).

### Experiments: real data (2D+3D)

- Freely moving human subject.
- 3D volume (384x192x16) acquired with FLASH sequence.
- TR=350ms, TE=40ms, FlipAngle=18°, Slice/Thickness=4mm.

- Monkey’s brain in fixation gel: controlled motion.
- 3D volume (384x192x16) acquired with FLASH sequence.
- TR=100ms, TE=62ms, FlipAngle=35°, Slice/Thickness=1.5mm

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**References**

